DIVISION OF THE HUMANITIES AND SOCIAL SCIENCES

CALIFORNIA INSTITUTE OF TECHNOLOGY

PASADENA, CALIFORNIA 91125

OPTIMAL RESOURCE MANAGEMENT UNDER CONDITIONS OF UNCERTAINTY: THE CASE OF AN OCEAN FISHERY

Tracy R. Lewis

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The optimal management of a natural resource under stochastic conditions is analyzed in general terms, and with specific application to the Eastern Pacific Yellowfin tuna fishery. Uncertainties about (1) the current and future size of the resource, and (2) the market value of the resource and the cost of extracting it, exist due to variations in economic and environmental conditions. A Markov Decision Process model of the resource is developed to find optimal policies that maximize the discounted stream of expected social returns from resource use. In addition, the model is used to answer these questions: How do optimal programs for allocating resources in a deterministic environment compare with optimal programs under stochastic conditions? Do different attitudes toward social risk bearing as regards variations in resource rents have an effect on optimal decision rules? What is the effect of increased uncertainty about resource prices, extraction costs, and resource growth and depletion rates on optimal programs.

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I. INTRODUCTION

For the most part, studies of optimal management policies for both renewable and nonrenewable resources have been done under deterministic conditions.[1] Generally these analyses assume a world in which all current and future demands, prices, and costs are known, in which the current reserve of the resource can be observed and measured exactly, in which environmental factors affecting the growth or deterioration of the resource are either unimportant or are perfectly predictable, and in which the entire time path of reserves and extraction rates can be calculated with certainty for a given program of resource management. In reality, of course, there is not only uncertainty regarding current and future resource prices, as well as the effects of environmental changes on resource stocks, but also there is often uncertainty about the existing supply of the resource available for extraction. Deterministic models have dominated the literature thus far not because the elements of uncertainty are unimportant or because they have gone unrecognized,[2] but because existing stochastic models are either not operable or too difficult to work with. The purpose of this study is to mitigate some of the deficiencies in the literature by introducing and analyzing a general model of resource management that readily incorporates various aspects of uncertainty.

A. Description of the Model

In our model the resource, whether it be a fishery, a mineral deposit, an oil reserve, etc., is controlled by a hypothetical social manager. It is assumed that the manager chooses the rate of extraction in each period to maximize the expected social utility of the stream of economic rents from the resource. Although we are interested in socially optimal behavior, the model is also appropriate for describing resource use for different market and allocation systems. Elements of uncertainty may be accommodated in the analysis in the following forms: (1) Uncertainties may exist about the current size of the resource either because of difficulties in observing and measuring the resource stock, as in the case of fisheries, or because of the possibilities of finding new reserves through exploration, as in the case of minerals and oil; (2) the market price of the resource may vary due to fluctuations in consumer demand and the availability of substitutes. The costs of extracting the resource may also

be random; (3) unpredictable changes in the environment may perturb the natural rate of growth or deterioration of the resource as well as the effective rate of depletion by man. For example, variations in the weather and the temperature of the water may have an effect on the natural growth rate of a fish population and the rate at which the fish are caught.

The optimizing technique for this analysis, developed by Howard (1960) is an application of dynamic programming to a discrete time and finite state and action Markovian process model. The dynamic structure of the resource extraction program is described in terms of a simple, one-period Markov process model with a finite number of <u>states</u>. In its most basic scalor form a state is simply a possible size of the resource stock; in more complicated vector forms a state might contain information on the size of the stock, the season of the year, prevailing economic and political conditions, etc. During a particular time interval, the program is in a certain state if it is described by the value of all the variables that define the state. A <u>state transition</u> occurs when its describing variables change from the values specified for one state to those specified for another. Movements from one state to another, described by the <u>transition probabilities</u> are random as a result of variations in environmental and socio-economic conditions affecting the natural growth and depletion of the resource. Thus, the transition probabilities in the simplest scalor form depend on the growth of the stock (which is important in the case of renewable resources), and on the rate of extraction.

With the states and transition probabilities fully specified, the manager regulates the use of the resource over time to maximize the expected social utility of the stream of future rents from the reserve. It is assumed that there are a finite number of possible extraction rates in each period for the manager to select from. This type of formulation results in a sequential optimization problem that is solved by dynamic programming.[3]

B. Analysis of the Model

The model is used to study the effects of uncertainty on optimal decision rules for the allocation of natural resources over time. In particular, we focus on the following questions: How do optimal programs for allocating resources in a deterministic environment compare with optimal programs derived under stochastic conditions? Do deterministic decision rules serve as a good approximation for optimal stochastic programs? How do different attitudes for risk bearing with regards to variations in resource rents affect optimal decision rules? Is the usual practice of representing the "riskiness" of a project in terms of the social discount rate appropriate for use in stochastic sequential maximization problems such as ours? What is the effect of increased uncertainty about consumer demand, and resource growth and depletion rates on optimal programs?

An analysis of the questions posed above are applied to a study of a specific renewable resource, the Eastern Pacific yellowfin tuna fishery. Because the resource can replenish itself, models of renewable resources generally are more complex than models of nonrenewable resources. Thus, although the study pertains to fisheries, our model is easily modified for analyzing nonrenewable resource problems.

Apart from demonstrating the use of our model, the purpose of this study is to generate some practical policy recommendations for the management of the Eastern Pacific yellowfin tuna fishery. The fishery is not only important as a food source, but it also provides incomes for fishermen from the United States, Canada, Japan, and several South and Central American countries.[4]

C. Plan of the Study

The plan for this paper is as follows: in Section II the optimal allocation of the fishery resource is described in terms of a finite state and action Markovian decision process. First, changes in the fishery stock as a function of natural growth and depletion are specified. The economic characteristics of the yellowfin tuna fishery are described and the problem of dealing with the risk in fishery rents is discussed.

Next the optimal allocation of the fishery resource is formulated as a discrete dynamic programming problem. A list of the various allocation programs to be considered under various conditions of uncertainty in the fishery is provided. Finally, certain limitations of the model are discussed and suggestions for extensions are made.

In Section III optimal allocation policies are presented that correspond to various conditions of uncertainty about the economic and biological processes in the fishery. The effects of increasing uncertainty in consumer demand and population growth rates on optimal allocation programs are assessed. Different attitudes for risk bearing are analyzed for their impact on optimal programs, and the prospects for being able to represent risk via the discount rate are examined. Areas requiring additional empirical research are identified. The paper is concluded with a brief summary and discussion of results.

II. MARKOV MODEL

A. Growth Characteristics of the Resource

The size of the fishery stock at some time $t+1$ equals the stock at time t enhanced by the amount of natural growth during that period minus the amount extracted by man. Thus we assume

$$X_{t+1} = X_t + \eta_{1t} \bar{\Delta} f(x_t) - \eta_{2t} \bar{\Delta} L_t \qquad (1)$$

where X_t = stock at time t (measured in physical units)

L_t = expected catch rate at time t

$f(X_t)$ = expected rate of change in the stock due to natural growth

$\bar{\Delta}$ = length of each time period

η_{1t}, η_{2t} = nonnegative multiplicative random variables, independently distributed through time with stationary density functions $h_1(\eta_1)$ and $h_2(\eta_2)$ and expected values $\mathcal{E}(\eta_1) = \mathcal{E}(\eta_2) = 1$.

With respect to ocean fisheries, η_{1t} represents fluctuations in water temperature and in the availability of preditors and prey that would alter the natural growth of the stock, and η_{2t} measures the effect of varying environmental conditions on the effective catch rate.[5]

Defining E_t as a composite input variable representing the capital and labor used in fishing at time t,

$$L_t = g(X_t, E_t) \qquad (2)$$

where $g(\ )$ is a production function for landing fish. The stock X_t enters production essentially as a capital input, which when combined with the variable input, E_t, yields a flow of resource consumption. We assume

$$\frac{\partial g}{\partial X} \geq 0; \quad \frac{\partial}{\partial X} \frac{\partial g}{\partial E} \geq 0 \qquad (3)$$

reflecting the increased difficulty of harvesting the resource as it becomes more scarce.

The Schaefer Stock Production model (see Schaefer [1957]), which is used for describing the population dynamics of the yellowfin tuna, provides us with specific functional forms for $f(X_t)$ and $g(X_t, E_t)$. According to a stochastic discrete time version of this model suggested by Pella and Tomlinson (1969)

$$f(X_t) = (a - bX_t)X_t \qquad a, b > 0 \tag{4}$$

$$L_t = g(X_t, E_t) = kX_tE_t \qquad k > 0 \tag{5}$$

$$X_{t+1} = X_t + [\eta_{1t}(a - bX_t)X_t - \eta_{2t}kX_tE_t]\bar{\Delta} \tag{6}$$

where $\bar{\Delta}$ = .1 year is the length of each time interval. In equation (4) the expected rate of natural growth increases for ($0 < X < a/2b$) is maximized at $x = a/2b$ (referred to as the maximum sustained yield population) and decreases for ($a/2b < X < a/b$) as depicted in Figure 1. The maximum sustainable population is given by $X = a/b$.

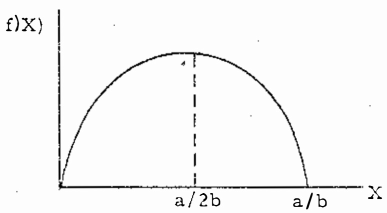

Figure 1. Population Growth Curve

Equation (5) describes a "mass contact" fishing technology where the catch rate is proportional to the physical contact between the fish and fishing effort, E. The constant, k, is called the "catchability coefficient" and is the percent of the total fish population removed

by one unit of effort. Equation (6) describes the changes in the population allowing for random variations in the growth and catch rates that are specified in equations (4) and (5) respectively. The distributions for η_1 and η_2 will be described in greater detail in Section II. E.

Estimates for the population parameters, a, b, and k, that are based on historical time series data for catches and effort, and were provided by the Inter-American Tropical Tuna Commission are listed in Table 4 on page 27. Details of the estimation procedure are described in Pella and Tomlinson (1969).

B. Economic Characteristics of the Resource

In each time period the flow of rent or net social returns from resource use is

$$R(X_t, E_t) = G(\eta_2 g(X_t, E_t)) - C(E_t) \tag{7}$$

where $G(\eta_2 g(X_t, E_t))$ = the total revenue and consumer surplus as a function of the actual amount of the resource harvested,

$C(E_t)$ = the total cost of E_t.

In general, changes in consumer demand, in the availability of substitutes, and in environmental conditions may cause the functions G( ) and C( ) to be stochastic. Consequently, $R(X_t, E_t)$ may vary with changes in the amount harvested (for a given E) and/or with changes in the revenue and cost functions. In the case of the tuna fishery we will allow for variations in price while assuming costs are nonrandom.

For Eastern Pacific yellowfin tuna we found prices were insensitive to the quantity of tuna purchased.[6] This is because the price of tuna is determined on the world market, and the quantity of yellowfin taken from the Eastern Pacific is only a small fraction of the total world supply of tuna. Consequently we assume

$$G(\eta_2 g(X_t, E_t)) = G(\eta_2 k X_t E_t) = p\eta_2 k X_t E_t \tag{8}$$

where p is the price per pound for unprocessed tuna.

We were unable to obtain the necessary cost data from boat owners in order to estimate $C(E_t)$.[7] Therefore we have assumed three hypothetical specifications for $C(E_t)$ enabling us to study the effect of different cost conditions on optimal resource use. The first of these specifications is

$$C(E_t) = 0 \tag{9a}$$

where the cost of effort is assumed to be zero. Although this specification is not representative of cost conditions in the yellowfin fishery, it is included here for general interest. Equation (9a) may be a good approximation for a sports fishing industry where people fish for the sake of enjoyment and relaxation.

The second specification is

$$C(E_t) = C_1 E_t + C_2 E_t^2; \; C_1 C_2 > 0 \tag{9b}$$

where costs increase more than proportionately with the amount of effort. This may result if the minimum earnings needed to attract

labor and capital at the margin rises as more of these inputs are employed in fishing, or if there are crowding and congestion externalities connected with large-scale fishing.

The third specification is

$$C(E_t) = C_3 E_t^{1/2}; \; C_3 > 0 \tag{9c}$$

where average and marginal costs decrease with greater allocations of effort. This may occur if there are economies of scale in the provision of fishing effort, or if there are gains in efficiency due to information sharing as the number of fishing vessels increases.[8]

In all of these specifications the fixed costs of effort are zero. The large purse seiners[9] that dominate fishing in the Eastern Pacific are quite mobile and can operate in numerous fisheries throughout the world. Because of the availability of other species in the same area, such as the skipjack, and the easy access to other fishing grounds, the fixed costs of fishing for yellowfin in the Eastern Pacific are minimal.

Combining the revenue and cost formulations in (8) and (9) we obtain three possible specifications for $R(X_t, E_t)$ given by

$$R(X_t, E_t) = \begin{cases} p\eta_2 k X_t E_t & (10a) \\ p\eta_2 k X_t E_t - C_1 E_t - C_2 E_t^2 & (10b) \\ p\eta_2 k X_t E_t - C_3 E_t^{1/2} & (10c) \end{cases}$$

each of which will be analyzed to determine the effects of different cost conditions in the fishery on optimal resources management.

C. Evaluation of Risks

The net social benefit from the use of a natural resource will depend on society's attitude toward variability in economic returns. The approach taken in this paper is that the risk attitude of individuals is important in determining risk preferences of society. It is generally accepted that individuals are not indifferent to risk and that investors must be paid a "risk premium" yield above the expected rate of return as compensation for the costs of risk bearing.

Accepting the fact that private risk aversion exists, the major issue with respect to evaluating public projects in general and the management of a resource in particular, is if the private cost of risk bearing represents a social cost as well. This will depend on the extent to which the returns from separate public projects can be "pooled" together (see Samuelson [1964] and Vickrey [1964]) and how extensively the risks from the project are spread among individuals in the economy (see Arrow and Lind [1970]).[10]

The actual distribution of risks and the attendant cost of risk bearing will depend largely on the structure of the management program. Economists have proposed numerous systems for resource management, including the imposition of taxes and subsidies on resource use, the sale of extraction licenses, and placing direct quotas and limitations on resource production. Each of these schemes results in a different distribution of the economic rents. A general theory of natural resource

allocation under uncertainty should allow for averse and neutral attitudes toward risk, depending on the institutional structure of the management program.

In selecting the optimal policy for resource use, we assume the manager chooses E_t to

$$\underset{E_t}{\text{maximize}} \quad \sum_{t=0}^{\infty} B^t \mathcal{E}U(R(X_t, E_t))\bar{\Delta}; \quad B = \frac{1}{1 + \rho}; \quad \rho > 0 \qquad (11)$$

where ρ is the riskless interest rate, and $\mathcal{E}U(R)$ is the certainty equivalence of the possible economic rents from the resource at time t. The social attitudes towards risk in resource rents is represented by the form of the social utility function $U(R_t)$. Strict concavity in the utility function implies risk aversion, while risk neutrality occurs if $U(R_t)$ is a linear function.

In taking this approach to resource management, we abstract from several issues which should be mentioned. First, the problems associated with "group decision making" are submerged behind our assumption of a single resource manager who makes allocation decisions for society.[11] Second, in maximizing the expected utility of the stream of rents from the resource we abstract from other possible goals of economic policy such as attaining high employment, acquiring a favorable balance of payments position, etc. While these are important issues pertaining to resource management, a proper treatment of these problems is beyond the scope of this study.

In analyzing management programs for the yellowfin tuna fishery two specifications of the utility functions are assumed:

$$U(R) = R \qquad (12a)$$

$$U(R) = \ln (R + G);\ G = 4.5 \times 10^{8}. \qquad (12b)$$

Due to the curvature of these functions (12a) reflects a risk neutral attitude and (12b) exhibits a risk averse attitude toward variability in the returns from the fishery. The natural log function in (12b) was chosen because it is easy to work with computationally. In stochastic models where variations in rents occur, it is possible for R to be negative. To insure that $\ln (R + G)$ exists, the constant G is specified to be large enough such that $R + G > 0$ for all possible values of R. The procedure for determining G is discussed in Lewis ([1975] pg. 102-103).

The two utility specifications in (12a)-(12b) combined with the various forms of the rent function we have presented in (10a)-(10c) yield the six classes of objective functions appearing in Table I. The analysis that follows will be carried through for each of these six classes. Each function is characterized by cost conditions in the fishery as well as the social attitudes toward risk bearing that exist.

Table 1. Classes of Objective Functions

Function	Class
pkX_tE_t	I
$pkX_tE_t - C_1E_t - C_2E_t^2$	II
$pkX_tE_t - C_3E_t^{1/2}$	III
$\ln(pkX_tE_t + G)$	IV
$\ln(pkX_tE_t - C_1E_t - C_2E_t^2 + G)$	V
$\ln(pkX_tE_t - C_3E_t^{1/2} + G)$	VI

D. A Discrete Markov Decision Process Model

The economic and biological characteristics of the fishery have been examined in Section II. A - II. C. Our next task is to develop a decision process for finding the optimal allocation program for the resource where programs are chosen to:

$$\underset{E_t}{\text{maximize}} \quad \sum_{t=0}^{\infty} B^t \mathcal{E}\, U[R(X_t, E_t)]\bar{\Delta}$$

$$\text{subject to:} \quad X_{t+1} = X_t + \bar{\Delta}[\eta_{1t}(a - bX_t)X_t - \eta_{2t}kX_tE_t]$$

Assume the resource is described by a finite number of states, X_i, for $i = 0, 1, 2, \ldots, 30$. Each state corresponds to a certain population size given by $X_i = i \times 10^7$ pounds. X_0 represents the minimum stock of 0 pounds, and X_{30} is the largest population. The parameter estimates of Table 4 indicate a maximum sustainable stock of 29.4×10^7 pounds so that $X_{30} = 30 \times 10^7$ pounds is large enough to include all probable values of the population. Since the Tropical Tuna Commission generally tries to maintain the stock at a level producing the maximum sustained yield, the current population is probably in the neighborhood of 15×10^7 pounds.[12]

In each time period, depending on the state of the resource, the resource manager can choose from among a finite number of possible effort allocations denoted by E_i^m. The "i" refers to the state occupied by the resource. The rate of effort is denoted by "m" where $m = 0, 1, 2, \ldots, M_i$ with $E_i^m < E_i^{m+1}$, and $E_i^{M_i}$ being the maximum effort expended in state i. These rates of effort are measured in terms of standardized boat days at sea per year and range in multiples of 250 from a minimum of 0 days to a maximum amount determined by economic conditions.[13]

The model we use for describing the population dynamics of the fishery is a "simple" or "one period" Markov process. The probability of making a transition to each state of the process depends on the state presently occupied and the management policy employed. A policy is a rule for selecting effort in each state. The transitions occur at regular discrete time intervals. Suppose at time t, $X_t = X_i$ for some i, and policy d is being utilized. For simplicity we will assume that policy d selects the "dth" effort rate in each state.

Then according to equation (6)

$$X_{t+1}^d = X_i + \bar{\Delta}\left[\eta_1 (a - bX_i)X_i - \eta_2 k X_i E_i^d\right] \text{ for all } i = 0, 1, 2, \ldots, 30 \tag{13}$$

By knowing the probability density functions for η_1 and η_2 and using equation (13) it is possible (see Lewis [1975], pp. 33-39) to calculate the transition probability that the resource moves from state i to state j under policy d; denoted by $p_{i,j}^d$ for all i, j, and d.

Let $V_i^d(N)$ be the expected social value from the resource obtained with policy d given that there are N periods left in the planning horizon and that the resource begins in state i. It is defined by

$$V_i^d(N) = \mathcal{E}\left[\sum_{t=0}^{N-1} B^t \left[U(R(E_t^d, X_t^d))\right]\bar{\Delta}\right] \text{for all } i = 0, 1, \ldots, 30 \tag{14}$$

given $X_0 = X_i$.

It is possible to rewrite equation (14) in recursive form such that

$$V_i^d(N) = q_i^d + B \sum_j p_{i,j}^d V_j^d(N-1) \text{ for all } i = 0, 1, \ldots, 30 \tag{15}$$

where $q_i^d = \mathcal{E}U(R(E_0^d, X_i^d))$ is the immediate expected social return from the resource. The values of q_i^d will depend on the functional form of U(R) (see Table 1) and the density functions for p and η_2. Equation (15) means that the expected social value, $V_i^d(N)$ is equal to the immediate expected return q_i^d plus the sum of discounted values of being in state j with N - 1 periods left, weighted by the probability that the resource will occupy state j in the next time period. Writing (15) in matrix form yields

$$V^d(N) = Q^d + BP^d V^d(N-1) \qquad (16)$$

where

$V^d(N)$ is a 31 x 1 column vector of the $V_i^d(N)$'s

Q^d is a 31 x 1 column vector of the q_i^d's

P^d is the Markov transition matrix corresponding to to policy d. $P^d = [p_{i,j}^d]$; i, j = 0, 1, 2, . . . , 30.

We want to consider a planning horizon of indefinite duration for the resource. It can be shown that (see Howard [1960] and Lewis [1975]) that

$$\lim_{N\to\infty} V^d(N) \equiv V^d = (I - BP^d)^{-1} Q^d \qquad (17)$$

In terms of this formulation the allocation problem for the resource manager is to find the policy that maximizes V^d. Ross ([1969], pp. 119-24) demonstrates that an optimal policy for this problem exists, that it is nonrandom and that the action it chooses depends only on the state of the process. Howard (1960) has developed an iterative scheme for finding the optimal policy which we have implemented in our study.[14] For a complete description of the computational techniques used to find optimal strategies, see Lewis (1975, pp. 104-10).

The model presented here is intended to approximate conditions in the real world where of course a continuum of states and policies exist.

A control theory model which assumes a continuous "state variable" (population) and a continuous "control variable" (effort allocations) was employed to derive optimal allocation rules for the fishery under deterministic conditions. These rules were compared with the optimal strategies obtained from the Markov decision model presented here to determine the effects of discretizing the state and control variables. The Markov decision model performed quite well in that the solution yielded by the programming and control theory methods were nearly identical (see Lewis [1975], Chapter III).

E. Specification of Cases

Stochastic Variation

The Markov model developed in Sections II. A - II. D will be used to analyze optimal programs for resource management under various conditions of uncertainty. The cases included in our study, listed in Table 2, are characterized by the kind of cost conditions for effort, by the types of stochastic variation in the economic and biological parameters, and by the social attitudes toward risk assumed in the model. For convenience we will refer to a particular program by the label corresponding to it in the table. The Roman numerals in each label refer to the specification of the utility and rent functions, reflecting the cost conditions and attitudes toward risk bearing that prevail in the fishery. The letters indicate the type of stochastic variation in the model, for which we assume there are four possibilities. In the yellowfin tuna and other fisheries[15] fluctuations in the consumer demand and changes in the availability of substitute food products cause the price of the resource to vary. To assess the impact of these variations on resource allocation we consider cases where price is random, denoted by the letter P in the labels appearing in Table 2. Because of changes in environmental conditions, the rate of fish landings (for a given allocation of effort), and the natural rate of growth of the stock may fluctuate. Therefore, we also include situations where the depletion rate is variable, denoted by $\underline{D}$; where depletion and growth rates are random and completely dependent, denoted by $\overline{\underline{DG}}$; and where they are independent, denoted by $\underline{DG}$. Summarizing, the programs listed in Table 2 are derived by permuting each of the six specifications of U(R) with the four kinds of stochastic variation in the model.

Table 2. List of Cases

Stochastic Variation / $U(R(X_t, E_t))$	Random Prices η_{1t}, η_{2t} Non Random	Random Depletion Rates η_{1t}, P_t Non Random	Random Growth and Depletion Rates (Complete Dependence) P_t Non Random	Random Growth and Depletion Rates (Complete Independence) P_t Non Random
$P_t \eta_{1t} kX_tE_t$	I-P	I-D	I-$\overline{DG}$	I-DG
$P_t \eta_{1t} kX_tE_t - C_1E_t - C_2E_t^2$	II-P	II-D	II-$\overline{DG}$	II-DG
$P_t \eta_{1t} kX_tE_t - C_3E_t^{1/2}$	III-P	III-D	III-$\overline{DG}$	III-DG
$Ln(G + P_t \eta_{1t} kX_tE_t)$	IV-P	IV-D	IV-$\overline{DG}$	IV-DG
$Ln(G + P_t \eta_{1t} kX_tE_t + C_1E_t + C_2E_t^2)$	V-P	V-D	V-$\overline{DG}$	V-DG
$Ln(G + P_t \eta_{1t} kX_tE_t - C_3E^{1/2})$	VI-P	VI-D	VI-$\overline{DG}$	VI-DG

Frequency Distributions for Random Variables

The frequency distributions for the random variables in our model, price, p_t, the depletion parameter, η_{2t}, and the growth parameter, η_{1t}, are not known because of a lack of data.[16] Therefore, we make the following assumptions: All random variables are distributed independently over time. For computational convenience, in the $\overline{DG}$ cases we assume η_1 and η_2 are identical. The expected value of the variables is equal to the value they assume under deterministic conditions with $\mathcal{E}(p_t) = \$0.15$, and $\mathcal{E}(\eta_1) = \mathcal{E}(\eta_2) = 1$.

We are able to simulate a rich variety of stochastic conditions in the fishery by assuming that the variables have either a truncated triangular or uniform frequency distribution. Like the uniform distribution, the truncated triangular distribution is completely determined by the specification of two range parameters, assuming the distribution mean is fixed. For instance, the distribution, $h(\eta_{1t})$, for η_{1t}, an example of which appears in Figure 2, is constructed in the following manner:

> For all values of d_1 and d_2, which specify the range of the distribution, with $0 \le d_1 < 1 < d_2$, the frequency function for η_{1t} is completely determined by the two conditions:

$$\mathcal{E}(\eta_{1t}) = 1 \tag{18a}$$

$$\left.\begin{array}{lll} h(d_1) = 0 & \text{if} & (1 - d_1) > (d_2 - 1) \\ \begin{matrix} h(d_1) \\ h(d_2) \end{matrix} = 0 & \text{if} & (1 - d_1) = (d_2 - 1) \\ h(d_2) = 0 & \text{if} & (1 - d_1) < (d_2 - 1) \end{array}\right\} \tag{18b}$$

The distribution is symmetric when $(1 - d_1) = (d_2 - 1)$ or skewed when the equality doesn't hold. Truncated triangular distributions for the other two parameters, p_t, and η_{2t} are constructed similarly. Hopefully, determining the effect on resource allocation for different distributions of p_t, η_{1t}, and η_{2t} will help to indicate what empirical information on these random variables is needed for fishery management.

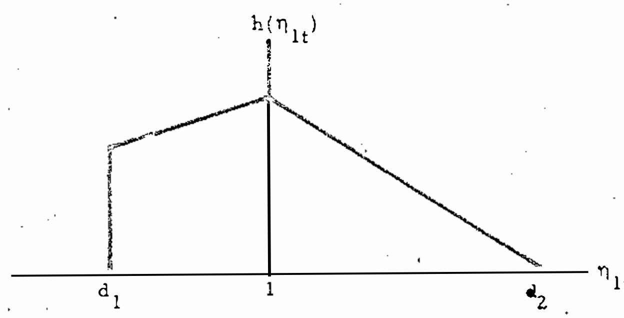

Figure 2. Example of a Typical Truncated Triangular Frequency Distribution

To identify a particular program, we will refer to it by the appropriate label in Table 2 and include a description of stochastic conditions that are simulated. Once the type of distribution is specified, only the "range parameters" are necessary to completely characterize the frequency function. For example, the label [I-D, Skd. L., (.3, 1.2)] denotes the case I-D where the distribution for the depletion parameter, η_{2t}, is truncated triangular and skewed to the left with η_{2t} ranging between 0.3 and 1.2. All distributions are truncated triangular, unless "U" appears in the description indicating a uniform distribution. Thus, [I-D, Sym. U., (.2, 1.8)] refers to the situation where the distribution for η_{2t} is symmetric and uniform with η_{2t} ranging between 0.2 and 1.8.

For the reader's convenience, a complete list and description of all the programs studied for each of the six specifications of the utility and rent functions appears in Table 3. The same set of cases were evaluated for all Classes I - VI of utility and rent functions. Three types of distributions, skewed left, skewed right, and symmetric are included in our analysis. We are interested in determining if the optimal consumption policies are sensitive to the different types of distributions. For each kind of distribution, the programs are arranged in increasing order according to the range of variation for the stochastic parameters. As we will see shortly, this allows us to study the effects of increased uncertainty on resource allocation.

Table 3. Program Listings Corresponding to Rent and Utility Specifications I-VI

Stochastic Variation / Distribution	Random Prices η_{1t}, η_{2t} Non Random	Random Depletion Rates η_{1t}, P_t Non Random	Random Growth and Depletion Rates (Complete Dependence) P_t Non Random	Random Growth and Depletion Rates (Complete Independence) P_t Non Random
Uniform	\$.15×(.8, 1.2)	(.8, 1.2)	(.8, 1.2)	(.8, 1.2)
Symmetric	\$.15×(.6, 1.4)	(.6, 1.4)	(.6, 1.4)	(.6, 1.4
(Sym. U.)	\$.15×(.4, 1.6)	(.4, 1.6)	(.4, 1.6)	(.4, 1.6)
	\$.15×(.2, 1.8)	(.2, 1.8)	(.2, 1.8)	(.2, 1.8)
Symmetric	\$.15×(.8, 1.2)	(.8, 1.2)	(.8, 1.2)	
(Sym.)	\$.15×(.6, 1.4)	(.6, 1.4)	(.6, 1.4)	
	\$.15×(.4, 1.6)	(.4, 1.6)	(.4, 1.6)	
	\$.15×(.2, 1.8)	(.2, 1.8)	(.2, 1.8)	
Skewed Left	\$.15×(.6, 1.2)	(.6, 1.2)	(.6, 1.2)	
(Skd. L.)	\$.15×(.4, 1.2)	(.4, 1.2)	(.4, 1.2)	
	\$.15×(.2, 1.2)	(.2, 1.2)	(.2, 1.2)	
Skewed Right	\$.15×(.8, 1.5)	(.8, 1.5)	(.8, 1.5)	
(Skd. R.)	\$.15×(.8, 1.7)	(.8, 1.7)	(.8, 1.7)	
	\$.15×(.8, 1.9)	(.8, 1.9)	(.8, 1.9)	

For DG and $\overline{DG}$ conditions, only one range of variation is indicated since we are assuming that η_{1t} and η_{2t} have the same distribution. Recall however, that η_{1t} and η_{2t} are identical for DG, and independently distributed for $\overline{DG}$. For DG programs, only uniform distributions are analyzed. The cost of simulating DG conditions is more than for the other cases because of the computational and memory storage demands on the computer in generating two independently distributed random variables. After reviewing the other cases we did not find that the optimal consumption program for the skewed and symmetric distributions were sufficiently different to warrant including both distributions in the DG programs.

Earlier, we stated that the probability density functions for the stochastic variables in our model, η_{1t}, η_{2t}, and p_t are not known. Except for observations on p_t, there is also no information about the range over which these parameters vary. For the purposes of our study, we assume that each of the parameters varies at its widest limits within a range of 0.2 and 1.8 its expected value. Although the price variation for some programs is greater than the fluctuations in price commonly observed for the yellowfin tuna fishery (see Lewis [1975], Appendix III-A), these cases are included in our analysis for general interest.

The values for the rest of the parameters in the model are listed in Table 4.

Table 4

Biological Parameters	
$a = 3.057$	
$b = 1.035 \times 10^{-8}$	
$k = 7.85 \times 10^{-5}$	
Economic Parameters	
$\bar{p} = \mathcal{E}(p) = \$0.15$	$B = 0.9906$
$C_1 = 5.0 \times 10^2$	$G = 4.5 \times 10^8$
$C_2 = 6.0 \times 10^{-2}$	$\overline{\Delta} = 0.1$
$C_3 = 1.0 \times 10^5$	

Values for the biological parameters were provided by the Inter-American Tropical Tuna Commission. For the economic parameters $\bar{p}$ is the mean price per pound for unprocessed yellowfin tuna during the 1966-1972 period, expressed in terms of 1956 dollars. Since information on the cost of effort was not available, programs corresponding to several sets of values for C_1, C_2, and C_3 (including those in Table 4) were analyzed and found to be qualitatively similar. The value for B corresponds to an annual discount rate of 10 percent.

F. Possible Extensions of the Markov Model

In the model presented in Sections II. A - II. D, we have deliberately abstracted from certain complexities in order to isolate the effects of uncertainty about prices and growth and depletion rates on optimal resource use. Nevertheless, our analysis is easily extended to accommodate other factors that presumably have an impact on resource allocation. Generally this involves expanding the dimensions of our state space.

For example, the rate of growth of some resources, and the costs and revenues from resource extraction may vary with the season of the year. To account for these changes in our model each state could be characterized by the size of the stock as well as the season of the year. Then depending on the state, a particular growth equation for that season could be used to measure stock changes, and season specific cost and revenue functions could be employed to calculate profits.[17]

Another possibility is that several resources may be related either physically or economically. The obvious example occurs in ocean fisheries where different species which compete for food to survive are also economically related as they are good substitutes for each other in consumption. Where a mutual dependence between resource exists, a joint management program is necessary to achieve efficient resource use. To accommodate this possibility in our model

each state could be described by a vector of the sizes of each of the related resource stocks. Then in each period the manager would choose the rate of extraction for each resource to maximize joint returns.[18]

The model can also be modified in other ways to account for difficulties in observing the actual size of the resource (an obvious problem with fisheries) and to capture time trends in certain economic, biological physical parameters of the system (see Lewis, pp. 39-45). As in the previous examples, these modifications are made by extending the dimension of the state space. In principle, any process can be modeled if the states are appropriately defined. However, there are limitations as to how elaborate the model can be made, since the computational costs of solving for optimal policies increases rapidly as the state space is expanded.

III. ALLOCATION POLICIES UNDER CONDITIONS OF UNCERTAINTY

A discussion of our results will be organized around a series of four questions concerning the effects of uncertainty on optimal allocation policies. Before turning to these questions we will first, characterize the optimal allocation strategies for Classes I-VI and second, discuss the sensitivity of optimal programs to different stochastic specifications.

A. Allocation Strategies

Depending on the cost conditions in the fishery, optimal policies are either of the steady-state variety or of the cyclical type. The literature on fishery management has dealt almost exclusively with steady-state policies. As we shall see, this is because the economic rents from the resource are usually assumed to be a concave function of effort as is the case with Classes I, II, IV, and V.

The notion of a steady state becomes obscured under stochastic conditions when the population is perturbed constantly by random variations in growth and depletion rates. However, looking at expected changes in the population we say that a steady-state fishing strategy is one for which the stock converges (on average, or ignoring variations in growth and depletion rates) to an equilibrium population size. The dynamics of the system are represented in Figure 3.

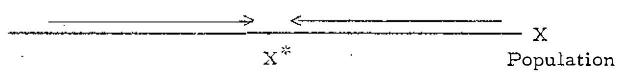

Figure 3

At X^* the expected natural growth of the population is exactly offset by the expected harvest in each period. X^* is the equilibrium population in that (on average) for populations X less (greater) than X^*

the stock is allowed to grow (is depleted) until $X = X^*$. As expected, X^* increases with larger values of the discount factor, B, since it is desirable to harvest less of the resource the higher society weights the consumption of future generations. With Classes II and V, X^* varies directly with C_2 as it becomes less profitable to fish for the resource as the cost of effort increases. This implies that X^* is larger for Class (II and V) programs than for Class (I and IV) programs. Our results indicate that the approach to equilibrium is more rapid for Class (I and IV) programs than it is for Class (II and V) programs.

In contrast to steady-state fishing, the optimal strategy for Classes III and VI is to fish "cyclically" in the following manner: for large populations, effort allocations are large causing rapid depletion of the stock. As the population decreases all fishing is stopped and the stock is allowed to grow until it reaches a sufficiently large size to begin the harvest once again. This type of "cyclical" fishing represented in Figure 4 takes advantage of the decreasing average cost of effort peculiar to Classes III and VI by employing large amounts of E_t whenever fish are harvested.

For larger values of C_3 the fishing cycles are more pronounced. The minimum population where fishing begins is larger along with the size of the harvest and the amount of effort employed. With larger values of C_3 the average cost of effort at all levels of E_t increases. Consequently to produce at the same average cost it is necessary to employ greater amounts of effort resulting in larger harvests and greater fluctuations in the population. With smaller values of C_3 the opposite occurs and fluctuations in the population are less pronounced.

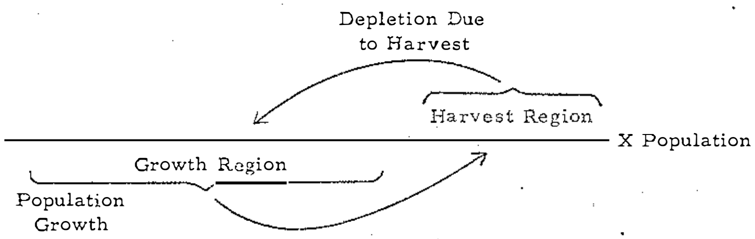

Figure 4

Most management programs are based on "steady state" fishing. The theoretical basis for steady-state fishing comes from the control theory analysis of optimal fishing behavior. These models only yield solutions for certain specifications of the rent function which, unfortunately preclude the possibility of economies of scale in supplying effort. Our results suggest that cyclical as opposed to steady-state fishing is optimal for situations where there are decreasing average costs of supplying effort. Since there has been little estimation of cost functions for various fisheries, the case for steady-state fishing may have arisen partially because of the analytical convenience of assuming convex cost functions for use in control theory analysis.

B. Sensitivity of Programs to Different Stochastic Specifications

A comparison of the optimal strategies corresponding to the programs analyzed in Table 2 indicates that:

Observation 1: Optimal effort allocations corresponding to variable depletion rate cases (specification D) are virtually unaffected by allowing simultaneous variation in the natural growth rate (specification DG and $\overline{DG}$).

Conceivably, for the parameter values of η_{1t} and η_{2t} we analyzed, fluctuations in the depletion rate dominate any effects of growth rate variation. Another explanation might be that random changes in the growth rates have no impact on allocation programs. To test these hypotheses we analyzed programs where only the growth rate was random for each of the six classes, assuming a variation for the parameter η_{1t} of (Sym. U. [.2, 1.8]). The optimal effort allocations for these programs differs only slightly from the deterministic program for each class. This tends to confirm our hypothesis that variations in the growth rate have a negligible effect on allocation programs.

Observation 2: The qualitative nature of our results is the same for all different parameter distributions we employ, whether they be symmetric, uniform symmetric, or skewed.

Although it does not appear that the type of distribution has a significant impact on resource allocation, we will see that the amount of variation in the random parameters has a pronounced effect on optimal decision rules.

C. Effects of Risk Bearing Attitudes on Optimal Harvesting Strategies

We are now ready to consider questions concerning the effects of uncertainty on optimal allocation policies.

Question 1: How do different attitudes toward social risk bearing, as regards variations in resource rents, affect optimal decision rules when uncertainty exists regarding the price of fishery products, the rate of depletion (for a given effort allocation), and the rate of population growth?

To answer Question 1, we contrast optimal decision rules for risk neutral (Class I-III) programs with risk averse (Class IV-VI) policies under identical stochastic conditions, for cases P, D, DG, and $\overline{DG}$. Our results are summarized in Observation 3.

Observation 3: For all variations, P, D, $\overline{DG}$, and DG listed in Table 3, a comparison of optimal risk averse programs with risk neutral policies indicates that:

a. At small populations the optimal allocation of effort and the resulting catch for risk averse programs are equal to or larger than the corresponding values for risk neutral policies.

b. At large populations the allocation of effort and the resulting catch for risk averse programs are generally less than the corresponding values for the risk neutral policies.

c. Except for Class III and VI policies, risk averse and risk neutral programs converge toward the same steady state population.

Sample comparisons between risk neutral and risk averse policies for conditions where depletion rates are random (D, Sym. U., [.6, 1.4]) appear in Figures 4-6.[19] Optimal policies are described by the effort to be allocated in each time period which depends only on the current population size. As a point of reference, levels of effort required to maintain the population at a steady state under deterministic conditions ($\eta_1 = \eta_2 = 1$) are traced out by the "Steady-State Effort Line."[20] Thus the population tends to increase (decrease) for effort levels lying below (above) the steady-state effort line.

Parts (a) and (b) of Observation 3 are explained by what we call the "concavity effect." The difference in effort allocation for risk averse and risk neutral policies is schematically represented in Figure 7. This difference measures the changes in effort caused by transforming utility from a linear into a concave function of R, or equivalently by changing Class I, II, and III functions to Class IV, V, and VI functions respectively.

In each period, the decision maker selects an allocation of effort based on the trade off between consuming a larger portion of the current stock, but at the expense of reducing the expected future stream of returns from the fishery. Roughly, the optimal policy is to limit current consumption for small stock sizes where the fishing conditions are poor in return for larger expected future revenues from the fishery once the population has increased. Of course for larger populations current consumption increases to take advantage of improved fishing conditions.

Because of decreasing marginal utility, the effect of making utility a concave function of R is to place greater weight on current consumption for small stock sizes where R_t is typically small, and less weight on consumption for larger populations where R_t is greater. This occurs because the addition to utility for a small increase in R_t is greater when returns are small, and vice versa. Another description of this effect, which we shall call the "concavity effect," is that it tends to moderate or smooth out the consumption policy as a function of stock size. That is, the difference in effort allocations and consumption rates corresponding to large and small population sizes is reduced by transforming the utility function. This effect is quite pronounced for Classes I and III as illustrated in Figures 4 and 6.

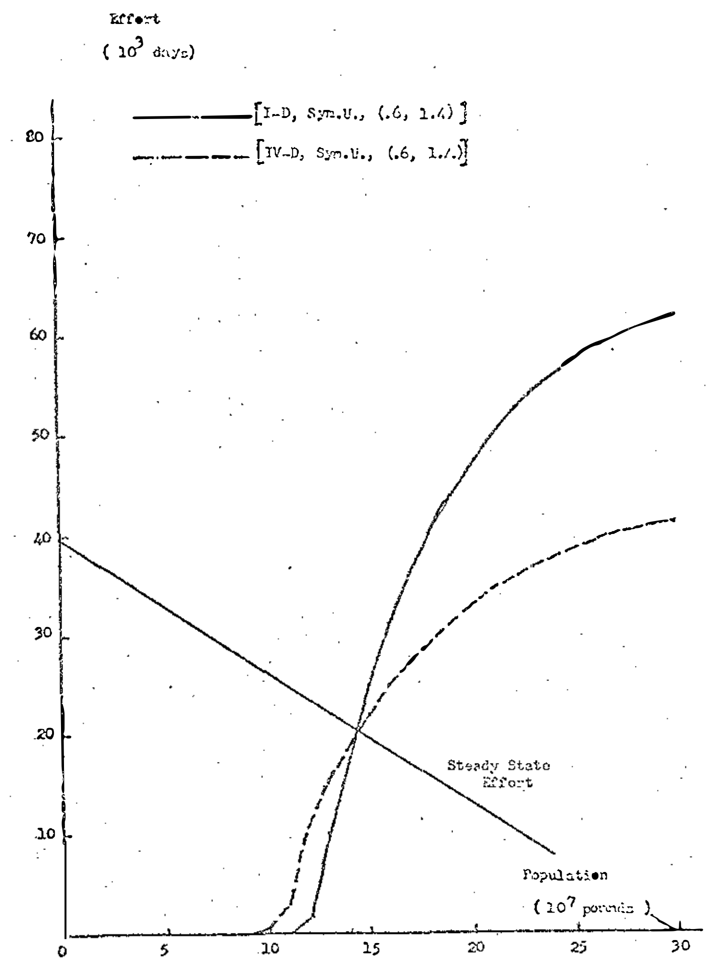

Figure 4. Comparison of Optimal Risk Averse and Risk Neutral Programs with Variable Depletion Rates for Classes I and IV

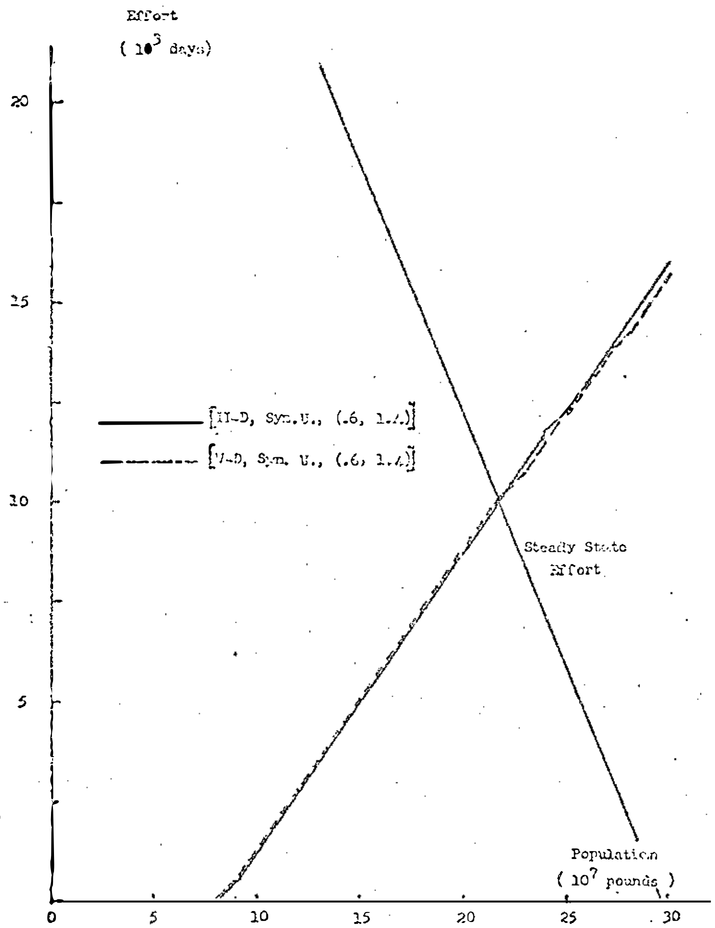

Figure 5. Comparison of Optimal Risk Averse and Risk Neutral Programs with Variable Depletion Rates for Classes II and V.[21]

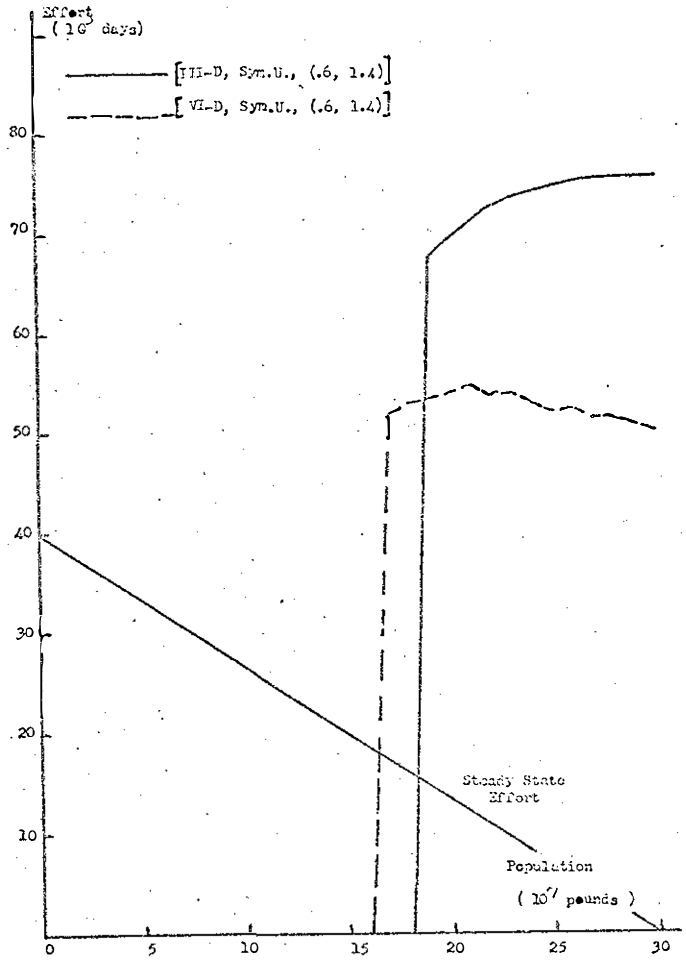

Figure 6. Comparison of Optimal Risk Averse and Risk Neutral Programs with Variable Depletion Rates for Classes III and VI

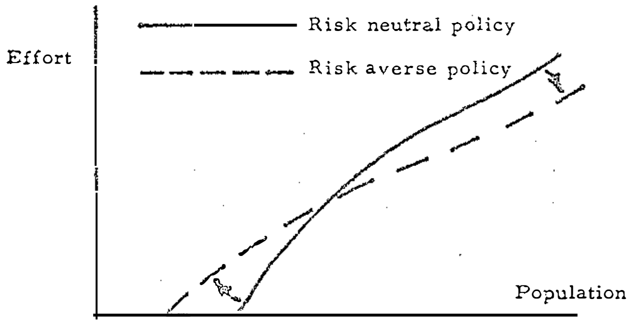

Figure 7. Schematic Difference between Risk Neutral and Risk Averse Policies

Notice that R_t is a strictly concave function of E_t for Class II. By arguments similar to those used above, we expect a more moderate consumption policy for II than for the other Classes, I and III. This is verified by examining Figures 4-6. In addition, the concavity effect will be less pronounced for Class II cases since R_t is already strictly concave in E_t.

According to Figures 4 and 5, Class (I and IV) and (II and V) programs tend to converge to the same steady-state population. Because of the concavity effect, risk averse programs generally converge to equilibrium at a slower rate than risk neutral policies.

For Class III and IV programs, as depicted in Figure 6, "cyclical" as opposed to "steady state" fishing is optimal. The cycles are less pronounced for risk averse policies than for risk neutral programs due to the concavity effect.

D. Effects of Increasing Uncertainty

Question 2: What is the effect on optimal consumption strategies for increased uncertainty regarding prices, and depletion and growth rates?

To answer this question we analyze changes in consumption policies for different distributions of prices, and growth and depletion rates. All these distributions belong to the class of mean-preserving spreads (the mean of the random variable is unchanged for all distributions), and are ordered according to how "risky" or "uncertain" they are. Adopting the definition of "increasing uncertainty" from Hadar and Russell (1969), Hanoch and Levy (1969), and Rothschild and Stiglitz (1970), we say that one distribution, f, is more uncertain than another, g, if

$$\int U(x)\, f(x)\, dx \le \int U(x)\, g(x)\, dx \qquad (19)$$

for all risk averters--those with concave utility functions, U. It can be shown[22] that (19) is formally equivalent to

$$T(Y) = \int_a^b (F(x) - G(x))\,dx, \quad T(Y) \ge 0 \text{ and } T(b) = 0 \qquad (20)$$

where F and G are the cumulative density functions corresponding to f and g, and it is assumed that the points of increase for F and G are contained in the closed interval [a, b].

Looking at Table 3 the distributions, symmetric uniform, symmetric, skewed right, and skewed left for each of the random parameters, p_t, η_{1t}, and η_{2t} are arranged according to the length of the interval over which the variable is allowed to range. It is easy to verify that according to condition (20) the distributions become more risky or uncertain as the range of variation for each of the parameters increase. For example, the distribution of price is more uncertain for [Sym. U., (.4, 1.6)] than it is for [Sym. U., (.6, 1.4)], and [Skd. L., (.4, 1.2)] is more uncertain than [Skd. L., (.6, 1.2)].

To sharpen our analysis we consider the effect of increasing uncertainty in two sections -- the first dealing with risk neutral social planners, and the second dealing with risk averse social maximizers. Within each section effects of increased variations in prices and increased uncertainty regarding depletion and growth rates are analyzed separately. From the results obtained here we can also compare resource use under deterministic conditions with resource allocation in a stochastic environment where uncertainty exists about prices, and growth and depletion rates.

Increased Variation in Prices, Growth and Depletion Rates -- Risk Neutral Social Planner

Variations in price have no effect on resource allocation for the risk neutral social maximizer as long as the expected price remains unchanged.

Changes in resource allocation for Classes I-III caused by increased uncertainty in growth and depletion rates are analyzed for all the cases of variations listed in Table 3 for D, DG, and $\overline{DG}$.

Observation 4: For the risk neutral maximizer the effect of increased variation in growth and depletion rates is characterized by the following comments:

a) Increasing variation in growth and depletion rates tend to decrease the optimal allocation of effort and the resultant catch corresponding to each population size.

b) This dampening effect on effort is greatest for Classes I and III rent functions. Within each class, the effect is more pronounced at the high end of the population scale where catches are typically large.

The effect of increasing uncertainty in the depletion rate on optimal programs for Classes I - III is represented in Figures 8 - 10.

Comparing the small variation cases [Sym. U., (.6, 1.4)] with the large variation situations [Sym. U., (.2, 1.8)] the change in effort increases with greater absolute variation in catch. This variation is proportional to the expected catch L, since $L_t = \eta_{2t} kX_tE_t = \eta_{2t}\bar{L}$.

As noted in Observation 4, and looking at Figures 8-10, we find that (1) the greatest effort adjustment occurs for Classes and III programs, and (2) within each class, changes in effort increase with population size. Note that the expected value and variation in

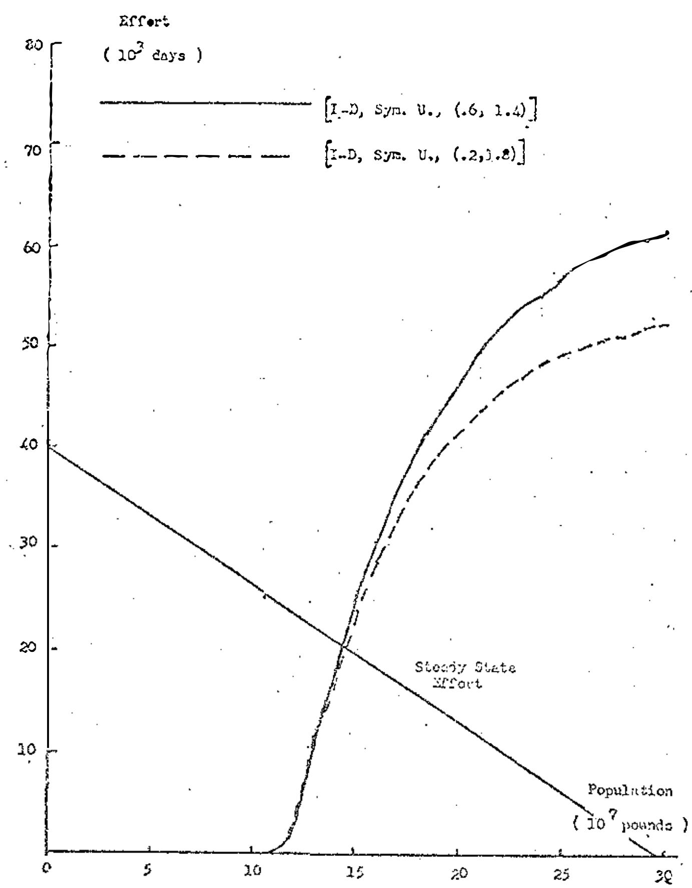

Figure 8. Comparison of Optimal Stochastic Programs with Increasing Variation in the Depletion Rate for Class I

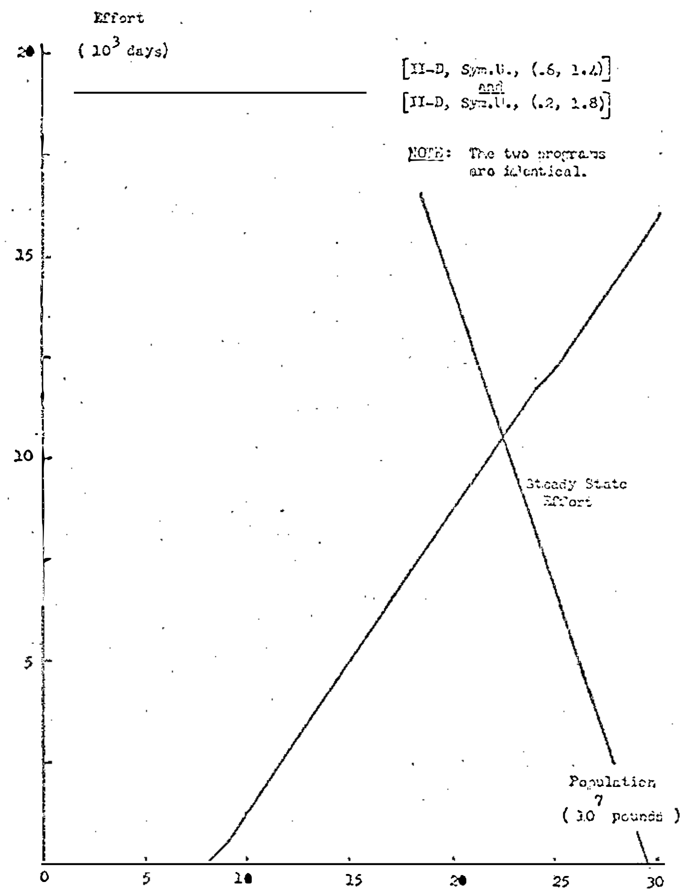

Figure 9. Comparison of Optimal Stochastic Programs with Increasing Variation in the Depletion Rate for Class II

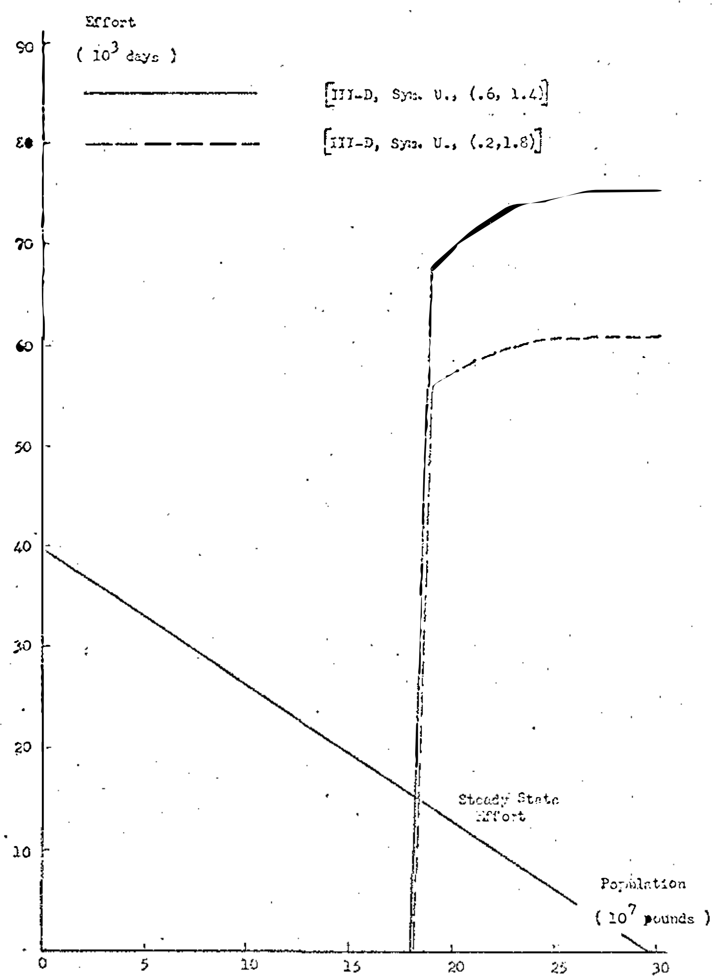

Figure 10. Comparison of Optimal Stochastic Programs with Increasing Variation in the Depletion Rate for Class III

catch is large for each of these situations. Generally L increases with population, and compared to Class II programs, the expected catch and the variation in L_t are greater for Classes I and III policies particularly at the upper end of the population scale.

Increased Variation in Prices, Growth and Depletion Rates--Risk Averse Social Planner

The effect of increasing variation in prices, growth and depletion rates on Class IV - VI programs are analyzed for all the P, D, $\overline{DG}$, and DG variations listed in Table 3.

Observation 5:

a. With increasing variation in prices or growth and depletion rates, the allocation of effort and resultant expected catch for small (large) populations are the same or increasing (decreasing) for Classes IV and V.

b. With increasing variation, the allocation of effort is generally decreasing and more evenly distributed over population states for Class VI.

c. The greatest change in effort caused by increasing uncertainty in prices or growth and depletion rates occurs for Classes IV and VI.

Changes in optimal programs for Classes IV - VI caused by increasing variation in depletion rates are presented in Figures 11-13.

A plausible explanation for our findings in Observation 5 is that the dispersion in rents, R_t, is proportional to the size of the catch when either p_t or η_{2t} are random. For all programs the optimal catch generally increases with population. Since the decision maker is averse to variations in R_t he tends to increase his catch for small populations, despite poor fishing conditions, since the dispersion in returns is smaller, and to decrease his catch at larger populations, because of the greater dispersion in returns.[23]

Effort allocations are distributed more evenly over population sizes for Classes II and V than for the other classes, due to the concavity of the rent function. Consequently, variations in the depletion rate, which tend to even out effort allocations, have less impact on Class V optimal programs.

E. Risk Adjusted Discounting

Question 3: Is it possible to account for the social attitudes towards risk bearing in the social discount rate?

This question is not to be confused with the issue of whether or not private costs of risk bearing represent social costs as well, and should therefore be taken into account in judging the desirability of public projects. Rather, our concern is with evaluating different analytical methods for representing risk aversion, assuming that the variability in fishery rents is a social cost that effects resource allocation decisions.

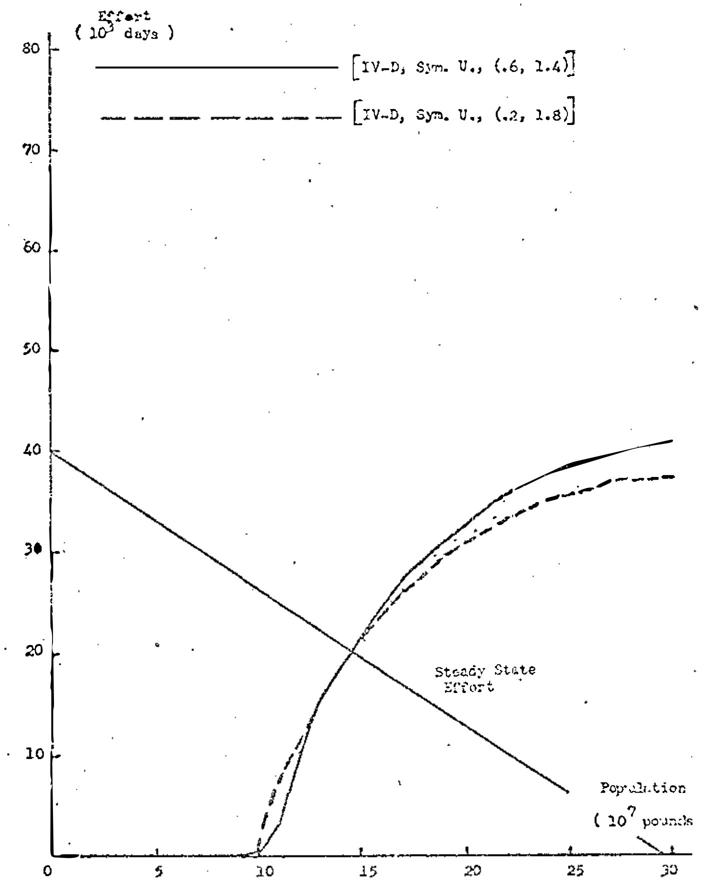

Figure 11. Comparison of Optimal Stochastic Programs with Increasing Variation in the Depletion Rate for Class IV

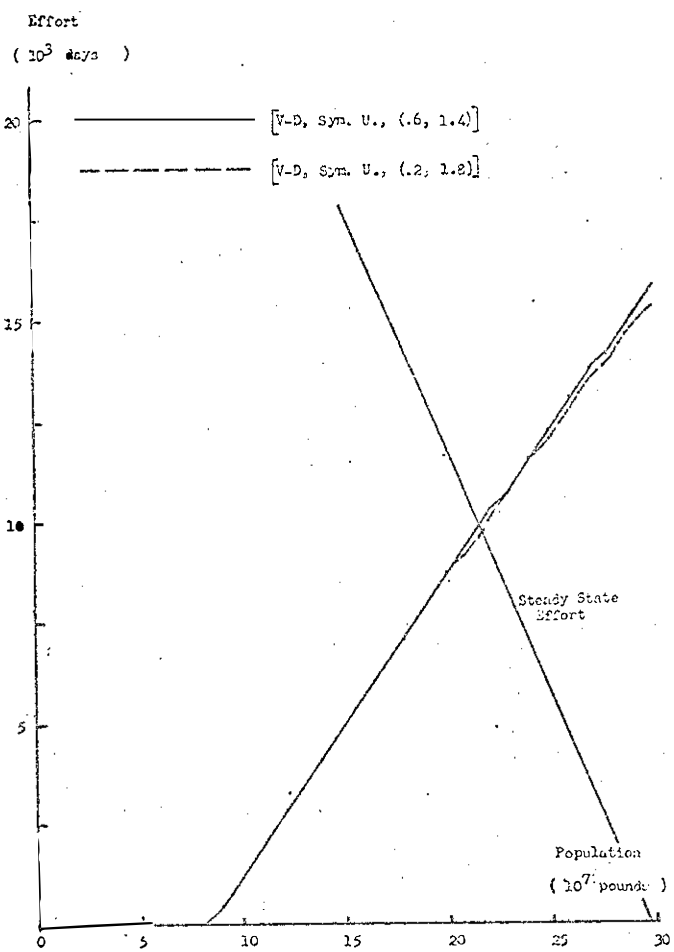

Figure 12. Comparison of Optimal Stochastic Programs with Increasing Variation in the Depletion Rate for Class V

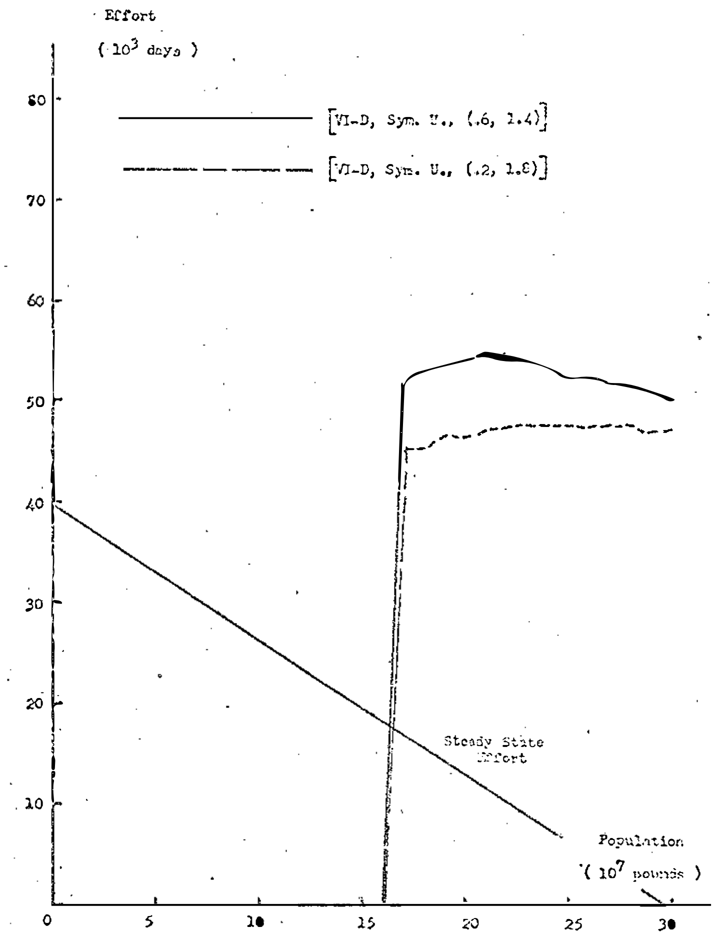

Figure 13. Comparison of Optimal Stochastic Programs with Increasing Variation in the Depletion Rate for Class VI

An alternative to capturing attitudes for risk bearing in the form of the utility function, is to employ a "risk adjusted" interest rate for discounting future uncertain returns. In this case the optimal consumption strategy is determined by choosing effort in each period to maximize the present value of fishery rents,

$$\sum_{t=0}^{\infty} B'^{t} \mathcal{E}[R(X_t, E_t)\bar{\Delta}] ; \quad B' = \frac{1}{1+\rho'} \tag{21}$$

where the rate of discount ρ' includes a "risk premium" yield over and above the "riskless" rate of interest ρ.

In the literature on cost-benefit analysis, by far the most common method of adjusting for risk is through the discount rate. Proponents of this procedure argue that the alternative of representing risk preferences with different forms of the utility functions requires direct knowledge of consumer's utility functions, and is therefore more difficult to implement.[24] However, the simplicity of the present value criteria in equation (21) is deceptive. In practice it is not easy to determine the correct value of ρ'. Several methods for calculating the social discount rate have been proposed, but all of them are difficult to implement.[25] However, a more serious objection to employing risk adjusted discounting is that risk is not a simple compounding function of time.[26] For example, in our model, variations in rent are independent of time and correspond to the size of the expected catch when there is uncertainty about prices or the rate of

depletion. Our general conclusion, stated formally in Observation 6 is that programs for exploitation of the fishery that are derived from maximizing the risk adjusted present value of returns, appearing in equation (21) are nonoptimal.

<u>Observation 6</u>: For each type of variation in our model, P, D, $\overline{DG}$, and DG, there does not exist a B' such that the solution to the problem

$$\max_{E_t} \sum_{t=0}^{\infty} B'^{t} \mathcal{E}[R(X_t, E_t)\bar{\Delta}]$$
$$\text{subject to } X_{t+1} = X_t + [\eta_{1t}(a - bX_t)X_t - \eta_{2t} k X_t E_t]\bar{\Delta} \tag{22}$$

yields a set of E_t's which are optimal for the problem

$$\max_{E_t} \sum_{t=0}^{\infty} B^{t} \mathcal{E}[U(R(X_t, E_t))]\bar{\Delta}$$

where $U(R(X_t, E_t)) = \ln(G + R(X_t, E_t))$.

We found that the effect of increasing the social discount rate ρ to account for increases in risk (equivalent to decreasing the discount factor, B), for all programs, regardless of the type of utility function or parameter variations involved, was to encourage a higher level of current consumption of the fishery resource for all population sizes. Intuitively, it seems natural for society to consume a larger portion of the resource currently as future felicities become less important. This is contrasted with the alternative convention of representing risk preferences in the form of the utility function. In this case current consumption increases or remains the same at small populations and decreases for large populations as variations in price, depletion and growth rates increase (see Observation 5).

Unfortunately, in our model, the simple approach of increasing the discount rate to capture risk is not operable. The reason for this is clear. Risk is not a simple compounding function of time and so no overall adjustment in the interest rate is suitable. Of course, there is nothing to preclude us from using a different rate for discounting returns in each period. The calculation of these rates would require knowledge of the variation in returns which is proportional to the expected catch in each period. However since this information is available, only after the optimal consumption programs have been determined, the use of different discount rates to adjust for risk is not practical.

The difficulties with trying to account for risk through the discount rate are of course not peculiar to our analysis. They occur whenever the risk associated with a particular project or activity is not a simple compounding function of time. The variation in net social returns for many consumption and production processes depends

on the level of the activity and not on the time during which it occurs. For these types of projects, the use of risk adjusted discounting is clearly inappropriate.

F. Policy Conclusions

Question 4: Based on the analysis of Questions 1-3 and our results concerning optimal resource allocation for the fishery in a stochastic environment, what practical policy recommendations can be made for the yellowfin tuna fishery? In particular, (1) how does the policy of maximizing the sustained physical yield from the fishery compare with optimal stochastic policies, (2) do the solutions to deterministic problems yield a sufficiently good approximation to the stochastic solutions to ignore probabilistic modeling all together, and (3) what additional information and data on the biological and economic processes of the fishery would be most useful for resource management?

Evaluation of Maximum Sustained Yield Policy under Stochastic Conditions

Several agencies, including the Tuna Commission advocate maximizing the sustained physical yield from the fishery. Critics of this policy assert that there is no particular utility in pursuing this program since the economic or social value of the resource is ignored. Despite these attacks, the policy is still retained by most fishery commissions because it is deemed to be "workable." The argument goes that if under the optimal economic program the stock converges to a steady-state size close to the maximum sustainable yield population, then the maximum yield policy should be adopted since it is probably easier for managers to follow and understand.

We find several difficulties with the maximum yield policy even as a workable program, when considered in a stochastic framework. With continual variations in the growth and the depletion rates, the population is rarely, if ever, in a steady state. The maximum sustained yield policy is deficient in that it abstracts from the non steady state or transient behavior of the fishery. However, even if the population tends to fluctuate around a certain stock size, as it does for Class I, II, IV, and V programs, this stock value will generally differ from the maximum sustained yield population. Of course, the concepts of steady state or maximum yield fishing are not applicable to Class III and VI programs since cyclical fishing is optimal.

Deterministic Results as an Approximation to Stochastic Solutions

Until only recently, economists in particular and social scientists in general, have avoided an explicit treatment of probabilistic models. Instead they have relied on deterministic results to

provide an approximation for stochastic solutions. The reasons for this are clear. When uncertainty is introduced into the analysis, the formulation of and solution to most problems becomes more involved and sometimes unmanageable. Finally, once the problem is resolved, the results of the stochastic model are often of a subtle and obscure nature and consequently are difficult to interpret for the policy maker. Besides this, there is the widely held belief that most conclusions of deterministic studies remain basically the same when a stochastic treatment is employed. Indeed, if the probabilistic answer to problems differs only slightly from the deterministic solution the large investment required for analyzing stochastic models may not be warranted.

However, regardless of the extent to which solutions differ, the adoption of stochastic methods is desirable if they effect an increase in the social returns from the resource that exceeds the attendant costs of research. Formalizing this notion we define the present value of the resource,

$$V^d(X_0) = \sum_{t=0} B^t \mathcal{E}[U(R(X_t, E^d(X_t)))] \overset{\Delta}{=}$$

to be the expected value of the sum of discounted utilities attainable from an initial population X_0 and following a policy denoted by 'd'. A policy is a rule or strategy for selecting an effort allocation depending on the size of population, such that $E_t^d = E_t^d(X_t)$. Assume 'D' is the optimal deterministic policy chosen to

$$\max_{d} \sum_{t=0}^{\infty} B^t [U(R(X_t, E_t^d(X_t)))] \bar{\Delta}$$

where price is non random, and (23)

$$X_{t+1} = X_t + [(a - bX_t)X_t - kX_t E_t] \bar{\Delta}$$

Let 'S' be the optimum stochastic strategy chosen to

$$\max_{d} \sum_{t=0}^{\infty} B^t \mathcal{E}[U(R(X_t, E_t^d(X_t)))] \bar{\Delta}$$

where price may vary, and (24)

$$X_{t+1} = X_t + [\eta_{1t}(a - bX_t)X_t - \eta_{2t} kX_t E_t] \bar{\Delta}$$

For a probabilistic environment, the increase in present value achieved by employing an optimal stochastic policy, S, rather than the deterministic consumption rule, D, is

$$V^S(X_0) - V^D(X_0) . \tag{25}$$

Obviously, we are not prepared to recommend whether or not stochastic modeling of the yellowfin tuna industry is warranted based on the type of cost-benefit criteria suggested above. Information on the empirical structure of the fishery is incomplete, and the programs we are investigating only simulate hypothetical situations in the fishery. Yet the following observations should provide us with a useful starting point for future policy analysis.

Observation 7: For the uncertainty programs P, D, $\overline{DG}$, and DG the increases in the present value of the resource for a given initial population realized by employing the optimal stochastic policy is

a) greatest for Classes I, III, IV, and VI and almost negligible for Classes II and V,

b) increases with larger initial populations and,

c) increases with greater variations in either price or the growth and depletion rates.

Examples of present value increases, calculated by equation (25) and corresponding to Classes I and III for variable depletion rate programs are plotted in Figures 14 and 15. The additions to present value for Class II programs are only of the order of magnitude of 10^3 dollars. Because our computer print outs were only designed to report numerical results up to five significant figures, the Class II figures are too small to report accurately. Similar computations for Classes IV - VI are not included here because they are more difficult to interpret, since present value figures are in terms of natural logs.

Looking at Figures 14 and 15 we see that present value increases are substantial for Classes I and III, that they increase according to the initial population size and that they become larger as the rate of depletion is more uncertain.[27] As a general result, it is not surprising to find that the magnitude of the present value increase depends on the extent to which optimal stochastic and deterministic policies differ. This explains the small increase for Classes II and V,

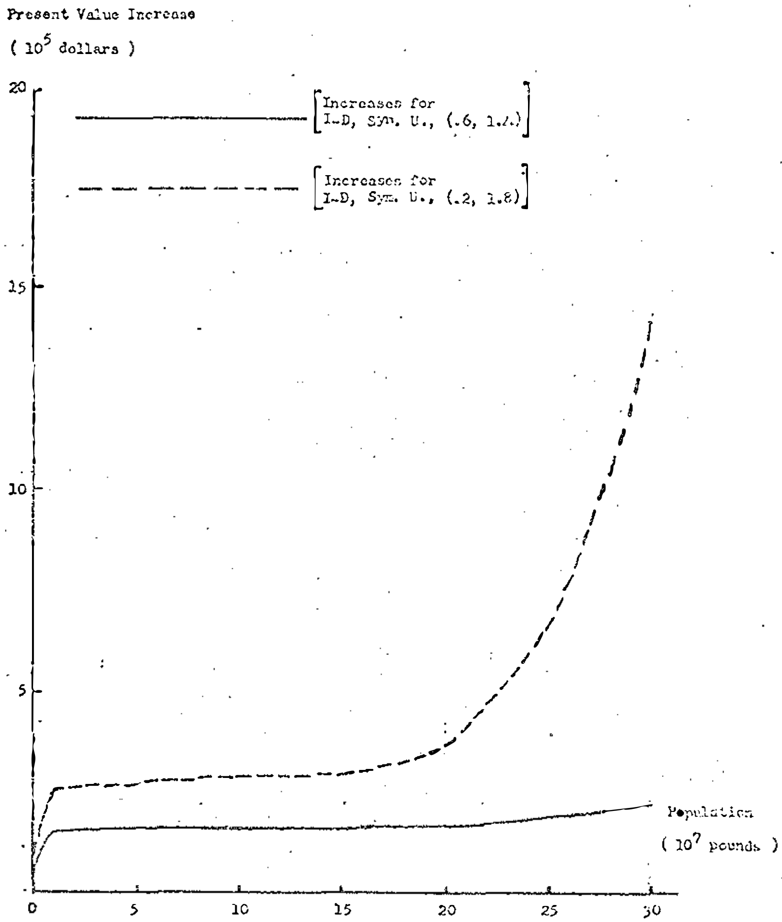

Figure 14. Present Value Increases for Class I Programs with Variable Depletion Rates

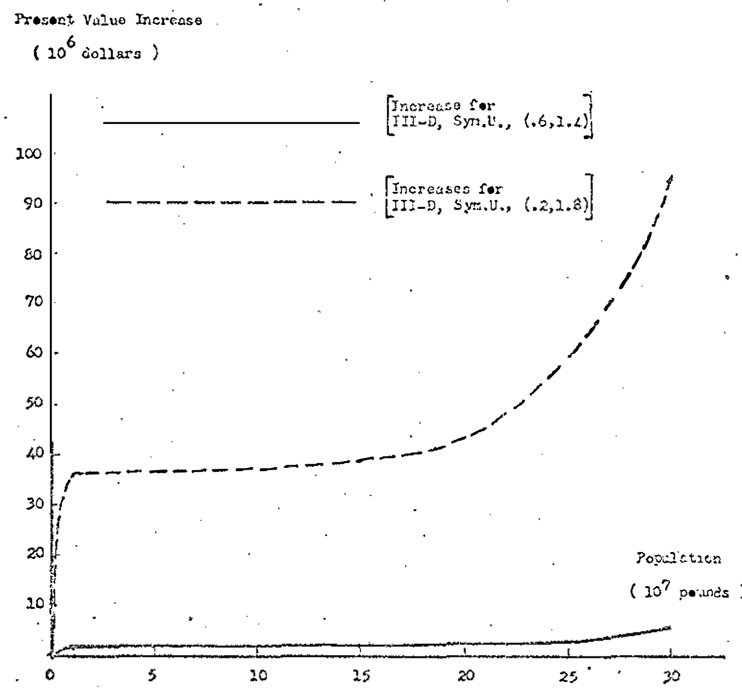

Figure 15. Present Value Increases for Class III Programs with Variable Depletion Rates

and that the increase in present values are larger for programs subject to more uncertainty.

At least for the range of parameter values we have analyzed, solutions to deterministic problems serve as excellent approximations for stochastic solutions in the Class II and V cases. On the other hand, our tentative conclusion for Classes I, III, IV, and VI is that a probabilistic treatment of the resource allocation problem is needed since deterministic consumption rules are poor substitutes for optimal stochastic strategies.

Suggestions for Additional Empirical Analysis

With our Markov model we have been able to assess the impact of various resource allocation programs on the fishery for a variety of different environmental and economic conditions. In effect, new policies and decision rules for operating the fishery have been tested under simulated conditions without running the risk of experimenting on the real systems. At the same time, our knowledge of the empirical structure of the fishery is only fragmentary. On the biological side, the nature of the variations in growth and depletion rates are as yet unknown, and on the economic side, information on the cost of fishing effort and the social attitudes toward risk bearing is incomplete.

Hopefully though, this study has yielded some valuable insights into which variables are more important than others in the

analysis of the fishery and which topics deserve the highest priority in future research endeavors. We have observed that optimal decision rules for operating the fishery are more sensitive to changes in certain variables and components of the model than others. This sort of information indicates the types of biological and economic data that will prove to be most useful for managing the fishery.

a. Biological Data

Without knowing the frequency functions for the rate of growth and depletion parameters, η_1 and η_2 we have assumed that they are distributed according to a uniform or triangular density function. If these serve as good approximations for the real distributions of η_1 and η_2 then our results indicate that we should be most concerned with gathering data to determine the range over which these parameters vary. We have observed that the optimal consumption strategies are sensitive to the amount of variations in these parameters. On the other hand, the type of distribution, whether it be uniform or triangular, symmetric or skewed does not seem to effect the optimal decision rules significantly.

b. Economic Research

Our results indicate that optimal allocation policies differ according to the specification of the cost of effort function. In estimating costs it will be particularly interesting to determine if marginal costs increase with greater allocations of effort, as is the case for Classes II and V. Under these conditions we observed that deterministic or certainty equivalent policies provide excellent approximations to optimal stochastic decision rules for the fishery.

One should realize that in gathering cost data, for our purposes effort is defined biologically in terms of efficiency units. It is a type of aggregate input which when applied to the fishery will remove or catch, on average, a certain percentage of the population. The components comprising a unit of effort need to be specified in order to estimate the quantity of capital goods and labor services used in the fishing process. Additionally, as with all cost estimations, some care is needed to insure that one is measuring the true opportunity cost of inputs rather than the accounting cost. This is particularly important in the yellowfin fishery, since the boats operating in this industry have a number of alternative opportunities for employment in other fisheries as well.

Optimal decision rules are also sensitive to society's attitudes toward risk bearing. Our approach has been to represent risk preferences in the form of the social welfare function as opposed to the more common, but as we argued less valid procedure of adjusting for risk through the social discount rate. The problem of actually estimating and providing a consistent representation of social risk preferences is a very difficult one, and we shall not attempt to resolve it here. The object of our study is much more modest: to determine the effect of different attitudes for risk bearing on optimal resource

allocation in the fishery. Beyond this, however, we have a few comments pertaining to the choice of the social welfare function.

Naturally the social attitudes towards variations in fishery rents will depend on how these rents are distributed among individuals in the economy, or for the case of an international fishery, how they are dispersed among the member countries. Consequently, implicit in the choice of a social utility function must be some provision for distribution of the fishery rents. At the same time, a number of regulatory schemes to prevent individuals from over exploiting the fishery have been proposed, including quota and licensing systems, each resulting in a different distribution of rents. This suggests that the two problems of: (1) formulating optimal consumption rules based on maximizing expected social utility and, (2) devising schemes to enforce these rules, must be solved simultaneously, as they are interrelated. The type of regulatory procedure will have an impact on distribution, which in turn will effect the choice of the social welfare function.[28]

Once the means for dividing the returns has been established the problem of choosing a welfare function that reflects social risk preferences still remains. The idea of constructing an aggregated social welfare function from a weighted sum of individual utility functions is perhaps theoretically possible,[29] but impractical. It appears to us that the choice might best be made politically. For example, using our Markov Decision model, one could determine various optimal allocation plans for the fishery derived by maximizing different social welfare criteria. These plans could then be reviewed and voted on by an electorate composed of individuals (countries), who were to receive a share of the rents from the fishery.

C. Summary Statements

The effect of uncertainty regarding prices, and growth and depletion rates on optimal allocation strategies have been analyzed for situations where society is averse and indifferent to variations in the returns from the fishery. The impact of uncertainty on consumption programs is directly related to the amount of variation in fishery rents determined by the size of the expected catch and the degree of fluctuation in the price and depletion rates. The variability of rents increases with population since the expected catch is typically greater. Key changes in optimal effort allocations occurring in a stochastic environment for different populations are of the following form:

For the risk neutral social planner, the allocation of effort and the resultant expected catch tend to remain constant or decrease as the price and growth and depletion rates become more uncertain. The largest changes in effort take place at the upper end of the population scale. The risk averse fishery manager increases effort at small populations to take advantage of the small fluctuations in rents, and decreases effort for larger populations to avoid greater risk in

returns. The effect of uncertainty on optimal Class II and V programs is relatively insignificant since the allocations of effort, expected catches, and thus the fluctuations in rents are small compared to the other classes.

The qualitative nature of our results are the same for all different forms of the frequency function analyzed. The most important element of the parameter distributions affecting allocation strategies is the range over which the variables are allowed to fluctuate. Optimal effort allocations corresponding to variable depletion rate cases are affected only slightly by allowing simultaneous variation in the natural growth rate.

The policy implications evolving from this analysis are: (1) The policy of maximizing the sustainable physical yield from the fishery, currently followed by the Tuna Commission, is not an efficient device for allocating resources under deterministic or stochastic conditions. The policy should be retained only if the political and social costs of switching to a new program are prohibitive. (2) Using the discount rate to capture risk is inappropriate since risk is not a simple compounding function of time and no overall adjustment in the interest rate is suitable. (3) Solutions to deterministic problems serve as excellent approximations for stochastic solutions in Class II and V cases. However, for the other classes a probabilistic treatment of the resource allocation problem is needed since deterministic consumption rules are poor substitutes for optimal stochastic strategies.

FOOTNOTES

1. The economic management of nonrenewable resources under deterministic conditions is discussed in Hotelling (1931), Gordon (1967), Herfindahl (1967), Scott (1967), Cummings and Burt (1969), Anderson (1972), Vousden (1973), Smith (1974), and Schmalensee (1975).

   The management of renewable resources is analyzed by Crutchfield and Zellner (1962), Plourde (1970) and (1971), Quirk and Smith (1970), Clark (1973), Spence (1973), Brown (1974) and Neher (1974).

   Notable exceptions are Heal and Dasgupta (1974) and Dasgupta and Stiglitz (1975) who consider the optimal exploitation of an exhaustible resource when there is uncertainty about the date when backup resources become available.

2. For example, see the discussion by Scott (1967, p. 26).

3. The most important feature of this programming approach is the ease with which elements of uncertainty are incorporated into the model. Stochastic elements are not accommodated at all in the control theory models used in the literature. For a more complete discussion of this point see Lewis (1975), p. 10.

4. The Eastern Pacific yellowfin tuna fishery is one of few international fisheries where the rate of fishing has been effectively controlled by a regulatory body, in this case the Inter-American Tropical Tuna Commission (IATTC). For management purposes IATTC collects and analyzes data on the fishery so that the yellowfin tuna is one of the most extensively studied populations in the world. Besides the obvious reason that tuna is a valuable resource, I decided to study this fishery because of the availability of reliable biological data.

5. In terms of mineral and petroleum reserves, η_{2t} might describe the effect on extraction rates for varying mining and drilling conditions. Normally, we assume $f(X_t) = 0$ for nonrenewable resources although nearly all minerals, natural gases and oils, generally conceived of being fixed in supply are replenishable given enough time.

6. See Lewis (1975), pp. 100-101.

7. The difficulties of obtaining costs of operation data from boat owners is discussed in Green and Broadhead (1964).

8. For example, Orbach (1975) notes that boats in different areas of the Eastern Pacific Tuna Fishery rely on each other for information about the location of schools of fish.

9. For a description of the purse seine fishing technology, see Green, Perrin, and Petrich (1971).

10. Arrow and Lind (1970) show that when the risks associated with any project are distributed among a large number of people so that the size of the share borne by each individual is a very small

component of his income, the total costs of risk bearing are negligible.

11. Conditions under which "group preferences" can be represented in the form appearing in equation (11) are discussed in Wilson (1968).

12. Recently the Commission has increased catch quotas to allow the population to reach a smaller size. This was done to obtain more information on the population dynamics of the stock.

13. Unless $E_i^{M_i}$ is given by technological or economic constraints, it is logically determined by the following restriction on the total catch during each time period. Clearly, the total catch cannot exceed the size of the stock available at the beginning of the period, i.e.

$$\bar{\Delta} k X_t E_t \leq X_t$$

implying that $E_t \leq \frac{1}{\bar{\Delta} k} = E_i^{M_i}$.

14. A linear programming approach, first suggested by Manne in "Linear Programming and Sequential Decisions," Management Science 6, no. 3, pp. 259-67 (1960), is also available for solving the Markov decision problem.

15. An important example where price fluctuations are quite large and are believed to have a significant impact on resource allocation occurs in the Peruvian Anchovy Fishery; see Segura (1972).

16. Unfortunately, the problem of insufficient data is not peculiar to the yellowfin tuna fishery. To my knowledge, at the present time there is little available information to estimate these frequency distributions for any of the ocean fisheries.

17. To my knowledge the effects of seasonal variation on the fishery have not been analyzed in the fishery economics literature. Despite its title, the paper by Bradley (1970) entitled "Some Seasonal Models of the Fishing Industry" does not deal with seasonal variation either.

18. Some interesting examples of multiple species models appear in Quirk and Smith (1969) and Lampe (1967).

19. For simplicity all results will be illustrated with examples of depletion programs assuming symmetric uniform distributions, although our observations apply to all the programs listed in Table 3.

20. In equilibrium $(a - bX)X = kXE$ or $E = \frac{(a - bX)}{K}$. Equilibrium values for E are represented by points on the "Steady State Effort" line in each figure.

21. Please note that the scale for effort on the vertical axis of all graphs corresponding to Classes II and V is enlarged.

22. See Rothschild and Stiglitz (1970).

23. This is partially offset by the fact that the utility function $U(R) = \ln(R + G)$ displays decreasing absolute risk aversion.

Thus as R increases, the resource manager should become less averse to risk.

24. For a discussion of this point see Hirshleifer and Shapiro (1970).

25. On this point see Baumol (1970) and Hirshleifer and Shapiro (1970).

26. See Prest and Turvey (1965).

27. Although the absolute increase in present value is large for Class I programs, the percentage increase is less than 1 percent. However, for the Class III programs the percentage increases range from 1 percent to 30 percent.

28. Another approach would be to consider the fishery as a private firm with shareholders. Recent results on the theory of the firm under uncertainty obtained by Ekern and Wilson (1974), Leland (1974), Radner (1974) and Forsythe (1975) suggest that maximizing the market value of the fishery might be appropriate even with incomplete markets for risk.

29. See Wilson (1968).

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